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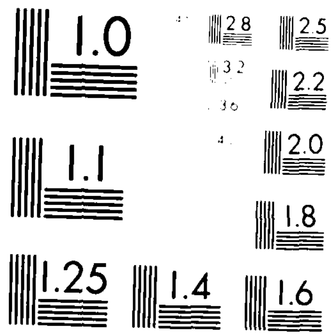

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EFFECTS OF MAGNETIC SHEAR ON LOWER HYBRID WAVES IN THE SUPRAURORAL REGION

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Effects of magnetic shear on lower hybrid modes are investigated. It is shown that due to non-local effects, even a small shear can significantly affect the instability, leading to stabilization for some parameter ranges. These results are of importance in the context of the recently proposed mechanism of lower hybrid acceleration and ion evolution in the suprauroral region.

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Abstract

Effects of magnetic shear on lower hybrid modes are investigated. It is shown that due to non-local effects, even a small shear can significantly affect the instability, leading to stabilization for some parameter ranges. These results are of importance in the context of the recently proposed mechanism of lower hybrid acceleration and ion evolution in the supraauroral region.

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I. Introduction.

It has recently been shown[1] that ions can be accelerated perpendicularly to the magnetic field by resonant interactions with current driven lower hybrid modes. This accelerated portion of the ions can evolve into conic distributions and propagate upwards along magnetic field lines. When they reach the region where they can be strongly energized by the electrostatic shocks, it is argued[1] that the resulting ion distribution can lead to the excitation of electrostatic ion cyclotron modes. This provides a plausible explanation of the simultaneous observation of the electrostatic ion cyclotron modes with the kev ion distributions in the suprauroral region.

Since the driving current also produces a magnetic shear, which generally exerts a stabilizing influence, it is quite important to investigate whether the lower hybrid mode remains unstable inspite of the influence of shear. The magnitude of the shear is generally quite small, with shear length L_s of the order of 500 km, and it may seem reasonable to ignore it altogether. However, our recent studies[2,3] in the context of the current driven ion cyclotron instability have shown that even a small shear can, due to non-local effects, sometimes produce a very significant reduction in the growth rate of an instability; under some conditions it even leads to stability.

Such an investigation was begun during the summer program and led to interesting preliminary results.[4] This effort was continued under the minigrant program and the results todate indicate that the effect of magnetic shear can be very significant, and can lead to a stabilization of the lower hybrid mode for a wide range of physical parameters of interest.

II. Scientific Background.

The importance of lower hybrid waves excited by energetic electron beams in the context of ion acceleration processes has recently been pointed out,[1] and it has been argued that a transfer of energy from the electrons to the ions is effected due to the simultaneous resonance of the lower hybrid (LH) mode with both the electron and ion populations. This ion acceleration process is particularly efficient at lower altitudes where the lower hybrid waves have a high intensity over a broadband of wavelengths. Typically, 1 eV ions can be raised in energy to hundreds of eV or beyond in the suprauroral region by the current-driven LH modes. Since the ions are energized primarily transverse to the field lines, they acquire "conic" distributions, with pitch angles clustering around 90° to 140°. Because of the mirror geometry of the earth's magnetic field, the transverse energy gained by the ions will be converted to longitudinal energy as they move upwards. When the ions propagate across a kilovolt electrostatic shock, they become field aligned. The combination of such keV ion distributions and of that of the background ions can lead to the excitation of electrostatic ion cyclotron modes. Thus, the LH acceleration mechanism may provide a possible explanation of the simultaneous observation of the electrostatic ion cyclotron modes with the keV ion distributions in the suprauroral region, particulary at altitudes above 5,000 km.

Central to this scenario is the idea that high intensity broad band LH waves are excited by the energetic electron bems. However, it should be noted that this conclusion is based on the simplified electrostatic dispersion relation,[1,5] and the effects of the magnetic shear generated by the field aligned currents have not been considered in equation (2) of Reference 1 or equation (1) of Reference 5. Magnetic shear generally exerts a stabilizing influence, and if it could effectively reduce the growth rates of the LH modes, the above mentioned explanations of the ion acceleration process and the generation of electrostatic ion cyclotron modes would become untenable. Thus recognizing the importance of investigating the influence of shear on the current-driven LH modes, we have initiated a systematic study of this problem.

We were motivated to examine this question inspite of the small strength of shear because of past experience and background in the context of our extensive study[2,3] of the current driven ion cyclotron instability, where we have shown that even a small shear can, due to non-local effects, produce a significant reduction in the growth rate and also reduce the band width of the wave numbers for which the mode is unstable.

The formulation of the problem of the lower hybrid mode in the presence of magnetic shear is given in the next section, followed by the method of solution and a discussion of results in subsequent sections.

III. Formulation of the Shear Problem.

For electrostatic waves, the dispersion relation for the current-driven lower hybrid mode is given by[1,5]

$$\frac{\omega_{pi}^2}{\omega^2} + \frac{k_\parallel^2}{k^2}\,\frac{\omega_{pe}^2}{\omega^2} = 1 - \frac{n_b}{n_o}\,\frac{\omega_{pe}^2}{k^2V_{tb}^2}\,Z'\left(\frac{\omega - k_\parallel U_{eb}}{k_\parallel V_{tb}}\right) \tag{1}$$

where the wave frequency ω obeys the condition $\omega_{ci} \ll \omega \ll \omega_{ce}$, ω_{ci} and ω_{ce} being the ion and electron cyclotron frequencies; $k^2 = k_\parallel^2 + k_\perp^2$; $k_\parallel \ll k_\perp$; $k_\perp r_e \ll 1$, r_e being the electron Larmor radius; $\omega \gg k_\parallel V_{eo}$, $\omega \gg k_\perp V_{io}$, where V_{eo} and V_{io} are the thermal velocities of the ambient electrons and ions; ω_{pi} and ω_{pe} are the respective plasma frequencies, n_b the electron beam density, n_o the ambient electron density, V_{tb} indicates the thermal spread of the beam around the beam velocity U_{eb}, and we have assumed $\omega_{pe}^2 \ll \omega_{ce}^2$.

The solution of the local dispersion relation (1) is given implicitly by

$$\omega^2 = \omega_{pi}^2\,(1 + \mu\theta^2) - (2\varepsilon/\alpha)\,[1 - y\,Z(-y)]\,\omega^2, \tag{2}$$

where $\quad \mu = (m_i/m_e), \qquad \theta = k_\parallel/k_\perp \approx k_\parallel/k,$

$$\alpha = (n_o/n_b), \qquad \varepsilon = \omega_{pe}^2/k^2V_{tb}^2 = (T_i/T_{eb})\,(N^2/k^2r_i^2)$$

$$N = \omega_{pi}/\omega_{ci}, \qquad T_{eb} = \frac{1}{2}\,m_e V_{tb}^2, \quad T_i = \frac{1}{2}\,m_i V_{io}^2;$$

and

$$y = u - \frac{\omega}{k_\parallel V_{tb}} = u - \frac{\varepsilon^{1/2}\Omega}{\psi} \tag{3}$$

with

$$\Omega = \omega/\omega_{pi}, \qquad \psi = \mu^{1/2}\theta, \qquad u = U_{eb}/V_{tb}. \tag{4}$$

For $(\beta/\alpha) << 1$, the local dispersion relation simplifies to

$$\omega_R = \{1 + \mu\theta^2\}^{1/2} = \{1 + \xi^2\}^{1/2} , \tag{5}$$

$$\gamma = \Omega_I = \pi^{1/2}(\beta/\alpha)ye^{-y^2}\ \Omega_R ,$$

where ω_R and γ denote respectively the real and imaginary parts of Ω. The growth rate γ varies with the angle of propagation θ, attaining a maximum when $y = y_0 = 2^{-1/2}$.

The <u>non-local</u> formulation is achieved[2,3] by introducting (i) $k_{\parallel} = k_y(x/L_s)$, where x is the distance measured perpendicular to the current sheet and L_s is the shear length and (ii) $k_x^2 = -\frac{d^2}{dx^2}$, which converts Eq. (1) into a differential equation. The magnetic field is along the z-direction and $k_\perp^2 = k_y^2 + k_x^2 \rightarrow k_y^2 - \frac{d^2}{dx^2}$. The resulting differential equation along with appropriate boundary conditions constitutes an eigenvalue problem for the (complex) frequency ω. With these substitutions, Eq. (1) can be rewritten as

$$\left(\frac{r_i}{L_s}\right)^2 \frac{d^2}{d\zeta^2} + Q(\omega,\zeta) \quad \phi = 0 \tag{6}$$

where, invoking $\zeta^2 << 1$, we have

$$Q = -(r_i k_y)^2 \{1+[1-\omega^2]^{-1} [\zeta^2-2(\beta/\alpha)\omega^2(1-yZ(-y))]\}. \tag{7}$$

The <u>local</u> dispersion relation is recovered from Eq. (7) by taking $L_s \rightarrow \infty$, i.e.

$$Q(\omega,\zeta) = 0 \tag{8}$$

which is easily verified to lead to Eqs. (2) and (5).

IV. Solution of the Shear Problem.

The departure from the local dispersion relation $Q = o$ can be investigated by solving the differential equation (7), subject to the boundary conditions that the electrostatic potential function remains bounded, or has outgoing energy boundary condition. In general the solution to the differential equation cannot be obtained in closed form, and numerical solution on a computer may be required. However, one can take advantage of the physical characteristics embodied in the Q function, and in particular, use a local Taylor expansion around some angle ψ_o, to be specified later, to obtain an approximate form for Q which enables one to obtain an analytical solution. This is the approach adopted here.

One can rewrite Eq. (7) in the form

$$\frac{\mu}{(k_y L_s)^2} \frac{d^2}{d\psi^2} + Q(\omega,\psi)\, \phi = o \quad , \tag{9}$$

$$Q(\omega,\psi) = -\{1 + (1-\omega^2)^{-1} [\psi^2 - 2(\mu/\omega)\omega^2(1-yZ(-y))]\}. \tag{10}$$

For small (μ/ω), Q is essentially a parabola and this suggests expanding Q upto second order around some convenient, as yet arbitrary "angle" ψ_o,

$$Q = Q_o + (\psi-\psi_o)\, Q'_o + \frac{1}{2}(\psi-\psi_o)^2\, Q_o''. \tag{11}$$

The Eq. (9) is a Weber equation, with a set of solutions given by the product of a Gaussian and Hermite polynomials and an eigenvalue equation for ω

$$Q_o = \mu^{1/2}(k_y L_s)^{-1}\,(2m+1)\,(-\frac{1}{2}\, Q_o'')^{1/2} + Q_o'^2/2Q_o'' \,. \tag{12}$$

For typical space parameters we have $k_y r_i \sim 1$, $L_s/r_i \sim 10^5$, $\mu \sim 10^{[illegible]}$ and then the first term on the right is generally negligible compared to the second. The dispersion equation then simplifies to

$$Q_0 = Q_0'^2/2Q_0'' . \tag{13}$$

This relation defines a complex ω for any given ψ_0. Different ψ_0 choices would give slightly different ω values. Since this approach rests on the validity of the expansion of Q as in Eq. (11), we must require ψ_0 to be in the region where the absolute value of the wave packet $|\phi|$ attains its maximum. For the mode $m = o$,

$$\phi = \exp\{-\frac{1}{2}(ik_y L_s)\lambda^{-1/2}(\psi-\psi_1)^2\} \tag{14}$$

where

$$\lambda = (-\frac{1}{2}Q_0'')^{1/2}, \qquad \mathrm{Re}\ \lambda > o \tag{15}$$

$$\psi_0 - \psi_1 = Q_0'/Q_0'' \tag{16}$$

and the condition that $\phi\phi^*$ attains its maximum at ψ_0 provides

$$\mathrm{Im}\ \psi_1 = o, \qquad \mathrm{Re}\ Q_0 > o. \tag{17}$$

Eqs. (13) and (17) together determine ω as well as ψ_0, and then the wave packet is given explicitly by Eqs. (14) to (16). When the first term in Eq. (12) is not negligible (e.g. with stronger shear, or large mass ratios) one must use the full Eq. (12) instead of Eq. (13), and Eq. (17) is replaced by

$$\mathrm{Im}\ (Q_0'/Q_0'') = 0, \quad \mathrm{Re}\ (Q_0'^2/Q_0'') > 0. \tag{18}$$

The width of the wavepacket, Eq. (14) is of the order of

$$\Delta\psi = \rho^{1/4}(k_y L_s)^{-1/2} \{Re(-\tfrac{1}{2}\omega_0'')\}^{1/4} [illegible]^{-1/2} . \qquad (19)$$

One of the requirements of self-consistency of this approach is that the cubic and higher order terms not included in Eq. (11) be smaller than the terms that are retained, with $(\psi - \psi_0) \sim \Delta\psi$.

In the weak shear limit, $L_s >>> r_i$, one can use the simpler Eqs. (13) and (17) and explicit evaluation of Q' and Q'' based on Eq. (10) leads to

$$[illegible] = [1-K]^{-1} , \qquad (20)$$

$$K = \frac{[illegible]}{(1+F_2)} \; \frac{\{F_2 - 2F_1 - (F_1^2/\alpha)\}}{(u-y)^2} + \frac{2\rho G}{\alpha} ,$$

$$F_1 = (u-y)^3 \; \{2y + (1-2y^2)\, Z(-y)\} ,$$

$$F_2 = -2F_1 + (u-y)^4 \; \{4(1-y^2) + (4y^3-6y)\, Z(-y)\} ,$$

$$[illegible] = y\, Z(-y) - 1$$

where [illegible], y and [illegible] are determined iteratively from Eqs. (3) and (20).

The condition Im ω_0 = 0 is equivalent to

$$Im \{[illegible]\psi_0^2 + (2\rho/\alpha)G]/K\} = 0 . \qquad (21)$$

A convenient procedure is to find the roots of (21) using y = u − [illegible] ψ_0^{-1} and then search near these roots to find the simultaneous solution of Eqs. (3), (20) and (21). The results obtained by this procedure are identical to the computer solution of Eqs. (13) and (17). These results are discussed in the next section.

V. [illegible] of Results.

[illegible] dispersion Eq. (13), subject to constraint Eq. (17), (or [illegible] Eq. [illegible] with constraint Eq. (21)) has been solved for the complex [illegible] for a wide range of parameters α, r and u. Typical beam strengths [illegible] $= \alpha^{-1}$ may range from 10^{-1} to 10^{-2}, characterized by the range [illegible]. We have covered in our computations a broader range [illegible] so as to include the asymptotic domains where scaling [illegible] can be predicted analytically. The beam velocity measured in units [illegible] spread, $u = U_{eb}/V_{tb}$ typically varies from $u = 1$ to 3. We [illegible] $u = 1$ to 5 in the tables given here and computation was carried [illegible] to reach the analytically accessible asymptotic domains. The beam [illegible] parameter $r = (\omega_{pe}^2/k^2V_{tb}^2) = (T_i/T_{eb})N^2/k^2r_i^2$, $N = \omega_{pi}/\omega_{ci}$, may range [illegible].

[illegible] describe the results for $\gamma = \mathrm{Im}\,\omega$ for the central ranges of [illegible], $u = 1$ to 5 for $r = 0.01$ and 0.1 respectively. [illegible] characteristics:

[illegible] range, there is only one mode, which is damped. The [illegible] with beam strength (i.e. with smaller α).

[illegible] for strong u and strong beam strength (in the [illegible] left hand corner of the Table). The magnitude of [illegible] some parameters give a growing mode (e.g. u = [illegible]). Thus the second mode is the dominant one where it [illegible].

[illegible] in beam strength leads to a domain where no normal [illegible] (e.g. u = [illegible] or smaller). Corresponding zones occur [illegible] at lower α. These are not covered by the range given in the [illegible]. The [illegible] thresholds are: $u = 4$, $\alpha = 8.33$; $u = 3$, $\alpha = 5$; $U = 2$, $\alpha = 2$; [illegible].

Table 1

$\beta = 0.01$

γ (in 10^{-4})

α \ u:	5	4	3	2	1
100	-0.57	-0.72	-0.72	-0.40	-0.11
80	-0.65	-0.83	-0.94	-0.60	-0.18
60	-0.76	-0.99	-1.25	-0.98	-0.31
40	-1.02	-1.28	-1.70	-1.80	-0.67
20	-2.05	-2.23	-2.73	-3.66	-2.44
	+0.018				
16.66	-2.58	-2.69	-3.15	-4.21	-3.35
	-0.15	+0.072			
12.5	-3.75	-3.73	-4.06	-5.19	-5.32
	-1.18	+0.022			
10	xxx	-4.80	-5.05	-6.15	-7.27
		-0.58	+0.063		

Table 2

$\beta = 0.1$

γ (in 10^{-4})

α \ u:	5	4	3	2	1
100	-5.73	-7.21	-7.31	-4.07	-1.15
80	-6.48	-8.39	-9.55	-6.08	-1.78
60	-7.68	-10.0	-12.6	-9.94	-3.11
40	-10.2	-12.8	-17.1	-18.3	-6.83
20	-20.9	-22.7	-27.6	-37.1	-25.0
	+0.18				
16.66	-26.6	-27.5	-32.0	-42.7	-34.5
	-1.56	+0.72			
12.5	-38.9	-38.5	-41.5	-52.8	-55.1
	-12.4	+0.22			
10	xxx	-49.7	-52.0	-62.8	-75.5
		-5.98	+0.62		

(4) For very large u, (e.g. u = 10), a fourth domain appears for strong beam strengths (α < 10). This is a growing mode with a significant γ, somewhat below the local growth rate. However this domain is too far away from the parameter region of practical interest. By extrapolation such a region might occur for moderate u values if we make α sufficiently small, below $\alpha = 1$. But $\alpha = 1$ corresponds to the case where the beam density is the same as the ambient electron density.

(5) For large α, (i.e. weak beam strengths) one can show analytically that γ scales as α^{-2}. This has been observed in the results for large $\alpha \sim 10^3$. The asymptotic scaling sets in for smaller α values for smaller u.

(6) γ is proportional to β in this range, as can be seen by comparing Tables 1 and 2.

From these observations we conclude that in the region of practical interest, there is only the damped mode which corresponds to the damped branch of the linear theory. The second mode (which is marginally positive or negative) occurs only for strong currents. Thus the current driven lower hybrid mode is essentially stabilized.

VI. Concluding Remarks.

We have formulated the problem of the current driven lower hybrid mode in the presence of shear and developed the necessary non-local treatment for its solution. Within the assumptions of the problem, we find that the mode is essentially stabilized. This is a significant result, especially in the context of the proposed ion acceleration mechanisms based on the lower hybrid instability. In view of the importance of the result, we comment briefly on the assumptions and the mode of solution, and indicate further avenues of extending this work.

We have started with a simplified, electrostatic local dispersion relation. The full electrostatic dispersion relation expressed as an infinite sum over the ion cyclotron harmonic terms should be employed as the starting point.

The current channel has been assumed to be uniform in space. The physical current sheets have finite widths (L_c) and the introduction of this new scale length can be expected to make the results a function of (L_c/L_s). For large enough L_c, one can expect the present results to emerge. On the other hand, for small L_c, the local theory will be recovered. The precise variation of γ with (L_c/L_s) would be an important study. We have carried out a similar study elsewhere[2] for the current driven ion cyclotron mode.

The method of solution employed here was based on using a local expansion of the ψ function. Direct numerical integration of the equation should be carried out (the so called shooting code method), to ascertain the accuracy of present results. Even within the analytical method, various self consistency checks indicate that the full Eq. (12) must be used in the strong current regime (i.e. large u, small α).

Finally, the possibility of travelling modes (in contrast to the normal modes studied here) should be examined, taking into account the practical geometrical features of the physical domain of interest.

We had to limit the scope of this work due to limitation of resources and we hope to pursue these and other related questions as and when further support becomes available. An interim account of part of this work was presented[6] at the 1983 Annual Meeting of the Plasma Physics Division of the American Physical Society.

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